Exercise 8

Given $f(x) = \frac{1}{x-4}$ and $g(x) = \frac{1}{6-x}$, find $f+g$, $f-g$, fg, and $\frac{f}{g}$. Determine the domain for each function in interval notation.

Solution

Determine each of the functions.

$$f+g = f(x)+g(x) = \frac{1}{x-4} + \frac{1}{6-x} = \frac{6-x}{(x-4)(6-x)} + \frac{x-4}{(x-4)(6-x)} = \frac{6-x+x-4}{(x-4)(6-x)} = \frac{2}{(x-4)(6-x)}$$

$$f-g = f(x)-g(x) = \frac{1}{x-4} - \frac{1}{6-x} = \frac{6-x}{(x-4)(6-x)} - \frac{x-4}{(x-4)(6-x)} = \frac{6-x-x+4}{(x-4)(6-x)} = \frac{10-2x}{(x-4)(6-x)}$$

$$fg = f(x)g(x) = \left(\frac{1}{x-4}\right)\left(\frac{1}{6-x}\right) = \frac{1}{(x-4)(6-x)}$$

$$\frac{f}{g} = \frac{f(x)}{g(x)} = \frac{\frac{1}{x-4}}{\frac{1}{6-x}} = \left(\frac{1}{x-4}\right)\left(\frac{6-x}{1}\right) = \frac{6-x}{x-4}$$

In each formula $x-4$ and $6-x$ appear in a denominator at some point, and since you can't divide by zero, $x-4 \neq 0$ and $6-x \neq 0$. Solve for x: $x \neq 4$ and $x \neq 6$. Therefore, the domain of $f+g$, $f-g$, fg, and f/g is $(-\infty, 4) \cup (4, 6) \cup (6, \infty)$.